Name: _____

Student number:

Teacher JNI DMH AMM

Newington College

Mathematics Extension 1
Trial Examination 2024

General Instructions

- Reading time: 10 minutes
- Working time: 2 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided at the back of this paper
- For questions in Section II, show relevant mathematical reasoning and/ or calculations
- Write your student number on the front of each exam writing booklet.
- Use a NEW writing booklet for each question. Extra booklets are available.

Total marks: 70

Section I - 10 marks (pages 2 —6)

- Attempt Questions 1–10
- Allow about 15 minutes for this section

Section II - 60 marks (pages 7 —13)

- Attempt Questions 11–14
- Allow about 1 hour and 45 minutes for this section

Results

Outcome/Section	Functions ME-F1 ME-F2	Permutations and Combinations ME-A1	Proof by Mathematical Induction ME-P1	Introduction to Vectors ME-V1	Trigonometry ME-T1 ME-T2 ME-T3	Calculus ME-C1 ME-C2 ME-C3	Total
Marks	/15	/3	/3	/19	/8	/22	/70

Section I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10.

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1 The parametric equations of a curve are given below.

$$\begin{aligned} x &= 4\cos\theta \\ y &= 4\sin\theta \end{aligned}$$

where $0 \leq \theta \leq \pi$.

What is the Cartesian equation of the curve?

A. $y = 1 - \dfrac{x^2}{16}$

B. $y = \sqrt{16 - x^2}$

C. $y = \dfrac{x^2}{16} + 1$

D. $y^2 = x^2 - 16$

2 Which of the following is equivalent to $2\cos 5x \sin x$?

A. $\cos 6x + \sin 4x$

B. $\cos 6x - \sin 4x$

C. $\sin 6x + \sin 4x$

D. $\sin 6x - \sin 4x$

3 Evaluate $\lim_{x \to 0} \frac{\sin 4x}{3x}$

A. 0

B. $\frac{3}{4}$

C. $\frac{4}{3}$

D. ∞

4 What is the size of the angle between the vectors $\underset{\sim}{a} = 3\underset{\sim}{i} + \underset{\sim}{j}$ and $\underset{\sim}{b} = 2\underset{\sim}{i} - \underset{\sim}{j}$?

A. $\frac{\pi}{6}$

B. $\frac{\pi}{4}$

C. $\frac{\pi}{3}$

D. $\frac{\pi}{2}$

5 A function is defined as $f(x) = \tan^{-1}(\tan(x))$.

What is the value of $f\left(\frac{9\pi}{4}\right)$?

A. $\frac{\pi}{4}$

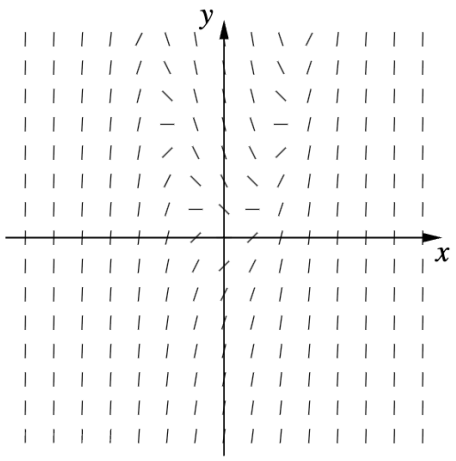
B. $\frac{5\pi}{4}$

C. $\frac{7\pi}{4}$

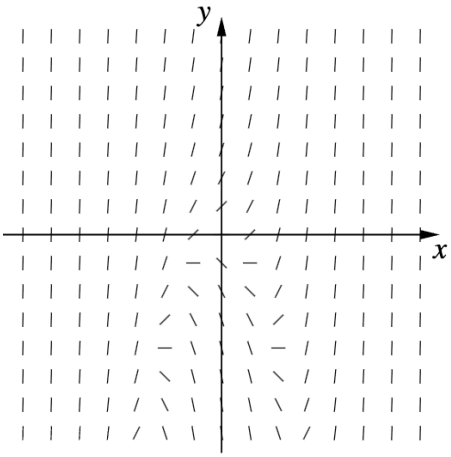
D. $\frac{9\pi}{4}$

6 Which of the following slope fields represents the differential equation $\frac{dy}{dx} = x^2 - y$?

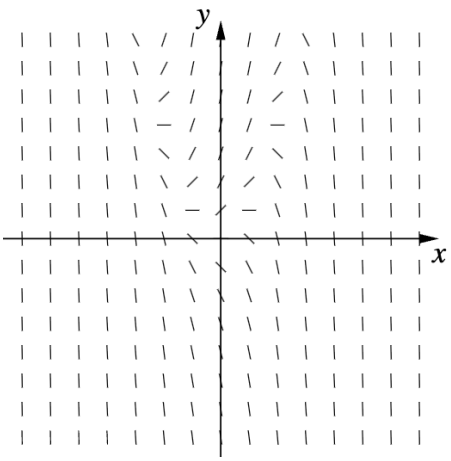
A.



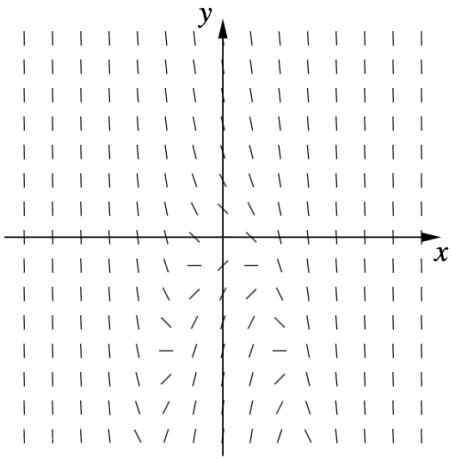
B.



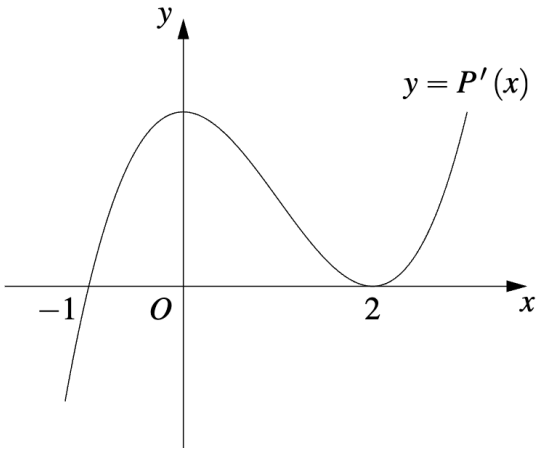
C.



D.



7 In the graph below, $y = P'(x)$ represents the first derivative of a polynomial $P(x)$ of degree 4, where $P(x)$ has a multiple root.



What can be said about the polynomial $P(x)$?

A. $x = -1$ is a root of multiplicity 3.

B. $x = 0$ is a root of multiplicity 2.

C. $x = 2$ is a root of multiplicity 2.

D. $x = 2$ is a root of multiplicity 3.

8 Sixteen people, consisting of eight doubles tennis partners, enter a local tennis competition.

Only four sets of partners are chosen to attend the celebration dinner. They are seated around a circular table.

In how many ways can the doubles partners be selected and seated around the circular table if the partners must sit together?

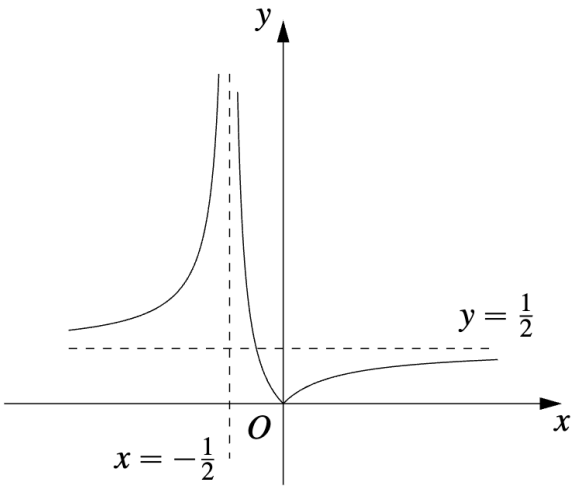
A. ${}^{8}C_{4} \times 3! \times (2!)^{3}$

B. ${}^{8}C_{4} \times 3! \times (2!)^{4}$

C. ${}^{16}C_{8} \times 3! \times (2!)^{3}$

D. ${}^{16}C_{8} \times 3! \times (2!)^{4}$

9 The diagram below shows the graph of the function $y = g(x)$, which is the result of a set of transformations on the graph of $y = f(x)$.



If $f(x) = \frac{1}{x-2} + 2$, which equation best represents $g(x)$?

A. $g(x) = \dfrac{1}{|\, f(x-2)\,|}$

B. $g(x) = |\, f(x-2)\,|$

C. $g(x) = \dfrac{1}{|\, f(x+2)\,|}$

D. $g(x) = |\, f(x+2)\,|$

10 Which statement is always true for the function $f(x) = \arcsin(x^2 + 2x + 1)$?

A. $f(x)$ has an inverse function in the domain $[-2,0]$.

B. $f(x)$ has an inverse function in the domain $[-1,0]$.

C. $f(x)$ has an inverse function in the domain $[-1,1]$.

D. $f(x)$ has an inverse function in the domain $[-2,2]$.

End of Section I

Section II

60 marks
Attempt Questions 11–14
Allow about 1 hour and 45 minutes for this section

Answer each question in the appropriate writing booklet. Extra exam writing booklets are available.

For questions in Section II, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use a SEPARATE writing booklet.

(a) A student representative committee of 7 people is to be chosen from a group of 10 men and 11 women. **2**

Find the probability that the committee is made of 3 men and 4 women.

(b) The polynomial $P(x) = x^3 - qx^2 + 32$ has real roots α, α and β. **3**

Find the value of q.

(c) Express $3\sin\theta + 4\cos\theta$ in the form $Rcos(\theta - \alpha)$, where $R > 0$ and $0^\circ \leq \alpha \leq 90^\circ$, correct to one decimal place. **2**

(d) Solve $\frac{x^2-6}{x} \leq 1$. **3**

(e) Solve $\int \frac{3}{1+4x^2}\,dx$. **2**

(f) Let $f(x)$ be a function where $f(2) = -11$ and $f'(2) = 8$. **3**

Find the equation of the tangent to the graph of $y = f^{-1}(x)$ at the point where $x = -11$.

End of Question 11

Question 12 (16 marks) Use a SEPARATE writing booklet.

(a) Use mathematical induction to prove that **3**

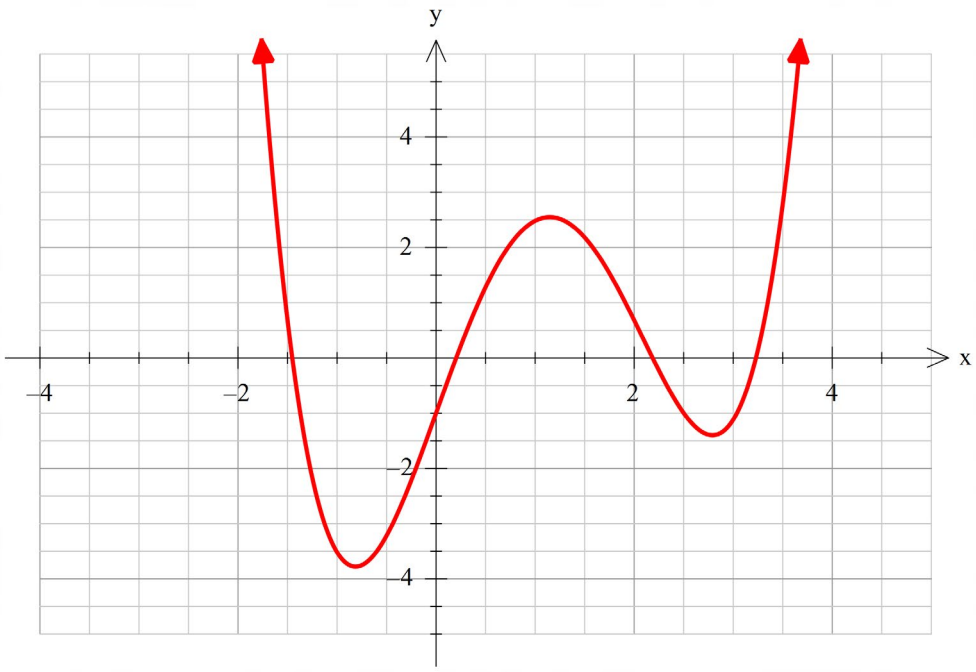
$$2^3 + 4^3 + 6^3 + \cdots + (2n)^3 = 2n^2(n+1)^2$$

for all integers $n \geq 1$.

(b) Evaluate $\int_3^{18} \frac{x}{\sqrt{x-2}}\,dx$ using the substitution $u = \sqrt{x-2}$. **3**

(c) Below is the graph of $f(x)$.

Using the separate sheet of paper provided, sketch the following graphs.



(i) $y = f(|x|)$ **1**

(ii) $y = \frac{1}{f(x)}$ **2**

Question 12 continues on the next page

(d) Let $S = 1 + 2\cos\theta + 2\cos 2\theta + 2\cos 3\theta$.

(i) Prove that $S \times \sin\frac{\theta}{2} = \sin\frac{7\theta}{2}$. **2**

(ii) Hence, show that if $\theta = \frac{2\pi}{7}$, then **1**

$$1 + 2\cos\theta + 2\cos 2\theta + 2\cos 3\theta = 0$$

(e) Find the volume generated when the area bounded by $y = 1 - \cos x$, $x = \frac{\pi}{4}$, $x = \frac{\pi}{2}$ and the x axis is rotated about the x axis. **4**

End of Question 12

Question 13 (15 marks) Use a SEPARATE writing booklet.

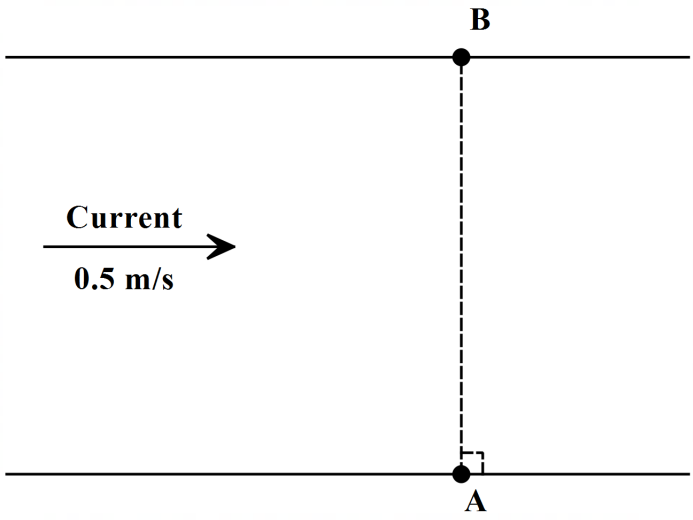
(a) The wind chill index, W, measures the apparent temperature in degrees by taking into account the speed of the wind, v km/h in a location. A meteorologist suggests that the wind chill index in the region where his laboratory is located is given by the equation

$$W = 19.5 - 7.4v^{0.17}$$

(i) Find $\frac{dW}{dv}$. **1**

(ii) Find the rate of change of W, given that the wind speed is 10 km/h and is increasing at a rate of 5 m/h per hour. **2**
Give your answer correct to two decimal places.

(b) In still water, Anthony can swim at 1.5 m/s. Anthony is at point A on the edge of a riverbank and considers point B directly opposite. A current is flowing from the left at a constant speed of 0.5 m/s.



For the following questions round your answers to 2 decimal places.

(i) If Anthony dives in straight towards B, and swims without allowing for the current, what will his actual speed and direction be? **2**

(ii) Anthony wants to swim directly across the canal to point B. At what angle should Anthony aim to swim in order that the current will correct his direction? **2**

Question 13 continues on the next page

(c) The points A, B and C have position vectors $\underset{\sim}{a}$, $\underset{\sim}{b}$ and $\underset{\sim}{c}$, respectively. Point D lies on the line segment AB, and has position vector $\underset{\sim}{d}$.

λ and μ are non-zero numbers such that $\lambda\underset{\sim}{a} + \mu\underset{\sim}{b} - \underset{\sim}{c} = 0$ and $\lambda + \mu = 1$.

(i) Show that the points A, B and C are collinear. **2**

(ii) It is known that $|\underset{\sim}{a}| = 2$, the angle between $\underset{\sim}{a}$ and $\underset{\sim}{b}$ is acute and, the area of triangle OAB is k square units. **3**

Show that $(\underset{\sim}{a} \cdot \underset{\sim}{b})^2 = 4(|\underset{\sim}{b}|^2 - k^2)$.

(iii) Given that $k = 6$, $|\underset{\sim}{b}| = 10$ and $\angle AOD = 90°$, find $\underset{\sim}{d}$ in terms of $\underset{\sim}{a}$ and $\underset{\sim}{b}$. **3**

End of Question 13

Question 14 (14 marks) Use a SEPARATE writing booklet.

(a) An invasive frog population of 1000 is introduced into Stanmore. The rate of the frog population is given by **2**

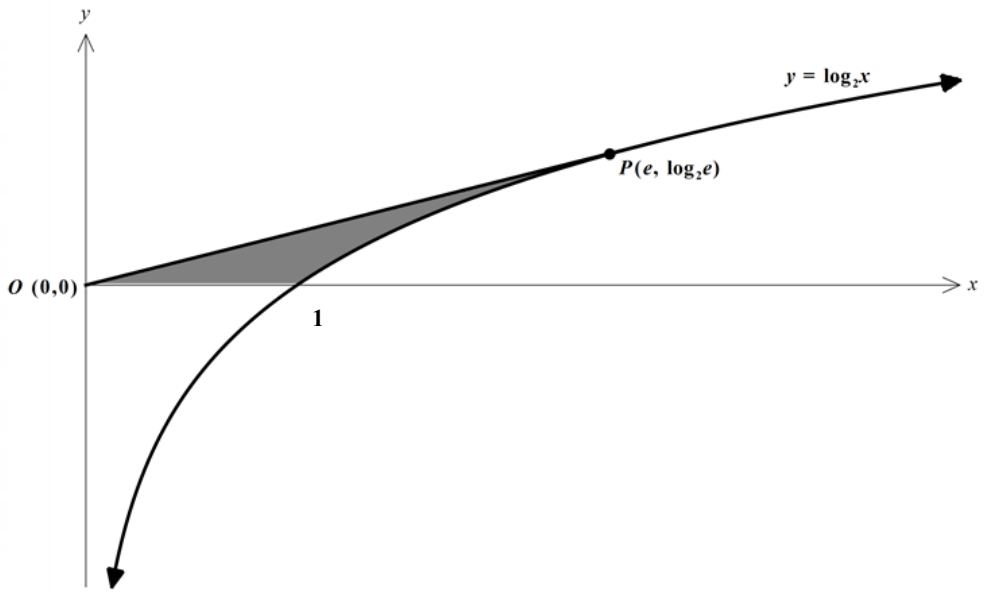
$$\frac{dP}{dt} = \frac{P}{20}\left(1 - \frac{P}{50000}\right)$$

where t is the time in months after the introduction of frogs.

By using the result $\frac{50000}{P(50000-P)} = \frac{1}{P} + \frac{1}{50000-P}$ (**do not prove this**) and solving the differential equation, show that

$$P = \frac{50000}{1 + 49e^{-\frac{t}{20}}}$$

(b) The diagram shows the point $P(e, \log_2 e)$ on the curve $y = \log_2 x$.



(i) Show that the line OP is tangent to the curve at the point P. **2**

(ii) The shaded region bounded by the curve $y = \log_2 x$, the line OP and the x axis is rotated through one revolution about the y axis. Find, in simplest exact form, the volume of the solid formed. **4**

Question 14 continues on the next page

Question 14 (continued)

(c) A stone is projected from point O with speed Vms^{-1} at an angle $\alpha = \tan^{-1}\left(\frac{4}{3}\right)$ above the horizontal. O is at the base of a ramp inclined at an angle $\beta = \tan^{-1}\left(\frac{1}{2}\right)$ above the horizontal. The stone moves in a vertical plane above the ramp under gravity where the acceleration due to gravity is g ms^{-2}. At time t seconds the position vector of the stone relative to O is $\underset{\sim}{r}(t) = (Vtcos\alpha)\underset{\sim}{i} + (Vtsin\alpha - \frac{1}{2}gt^2)\underset{\sim}{j}$.

(i) Show that the stone hits the ramp after time $T = \frac{V}{g}$ seconds. **3**

(ii) Use an appropriate trigonometric identity to show exactly that when the stone hits the ramp its direction of motion is inclined at an angle $\frac{\pi}{4}$ to the ramp. **3**

End of Question 14

End of paper